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Gauge fixing of Chern–Simons N-extended supergravity

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Abstract. We treat N-extended supergravity in $2+1$ space-time dimensions as a Yang–Mills gauge field with Chern–Simons action associated to the N-extended Poincaré supergroup. We fix the gauge of this theory within the Batalin–Vilkovisky scheme.

1 Introduction

Since Einstein achieved his theory, gravity has been treated as a particular geometry of space-time. The metric is what determines this geometry which, in the first order formalism, is described by the vierbein – or more precisely the dreibein in the context of the present paper – and a Lorentz connection. The dreibein components form a basis of the tangent vector space, equiped with a Lorentz structure. The metric is then derived from the dreibein. The Lorentz connection, considered as an independent field, turns out to be a functional of the dreibein by virtue of the field equations. In this case the action is that of Einstein–Palatini. N-extended supergravity is a supersymmetric generalization of gravity with N supersymmety generators. We shall be interested in N-extended supergravity in 3-dimensional space-time, a general presentation of which including conformal N-extended supergravity having been given in [1].

Witten [2] described gravity in $2+1$ dimensions as a gauge theory, writing the gravity fields as a Yang–Mills field with a Chern–Simons action and showing the equivalence of the Chern–Simons and Einstein–Palatini actions. The gauge group of this theory is that of Poincaré, which may be extended to the de Sitter or anti-de Sitter groups, corresponding to a positive or negative, cosmological constant, respectively.

The equivalence of 3-dimensional gravity with a Chern–Simons theory can be generalized to the case of N-extended supergravity, taking the Poincaré, de Sitter or anti-de Sitter supergroup as gauge group [3,4]. All one needs is an invariant quadratic form for these supergroups from which one can construct the Chern–Simons action, the latter being then interpreted as the N-extended supergravity action. The author of [4] did this for the anti-de Sitter supergroup considered as the product of two ortho-simplectic supergroups:

$$AdS(p,q) = OSP_+(p,2;\Re) \times OSP_-(q,2;\Re)$$

(where $p+q=N$ is the total number of supersymmetry generators). They considered various limiting cases, in particular the super-Poincaré limit of vanishing cosmological constant.

Our purpose is to implement a convenient gauge fixing of this N-extended super-Chern–Simons theory, taking into account the existence of a local vector supersymmetry generally associated to diffeomorphism invariance as usual in such topological theories [5]. Because of the latter invariance the complete gauge algebra closes only on-shell. We shall use therefore the Batalin–Vilkovisky version of the BRST gauge fixing scheme [7]. In order to obtain the gauge fixed action in a concise way we shall use a formalism of extended fields, i.e. superpositions of forms of all possible degrees [8,10].

We choose to work in the present paper directly in the super-Poincaré limit of zero cosmological constant. The construction for super-de Sitter with non-vanishing cosmological constant is similar [9] and will not be explicitly treated here.

The plan of this paper is the following. Section 2 reviews the construction of 3-dimensional N-extended supergravity as a Chern–Simons theory following [4] and then analyses all the gauge invariances, putting them together in a BRST operator and writing the corresponding Slavnov–Taylor identity. The gauge fixing is performed in Sect. 2.2. The paper ends with a concluding section.

2 N-extended supergravity

In order to construct an N-extended supergravity theory à la Chern–Simons in 3-dimensional space-time, with zero

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cosmological constant, we choose the N-extended Poincaré supergroup as a gauge group, whose Lie superalgebra is[1]:

$$
\begin{aligned}
&[J^a, J^b] = \epsilon^{abc} J_c, \quad [P^a, P^b] = 0, \\
&[J^a, P^b] = \epsilon^{abc} P_c, \quad [J^a, Q^I_\alpha] = -\frac{1}{2}(\gamma^a)_\alpha^\beta Q^I_\beta, \\
&[P^a, Q^I_\alpha] = 0, \quad [Q^I_\alpha, Q^J_\beta] = \delta^{IJ} (\gamma_a)_{\alpha\beta} P^a .
\end{aligned}
\tag{2.1}
$$

P^a, J^a and Q^I_α are the generators of space-time translations, Lorentz transformations and supersymmetry transformations, respectively, the Q^I's being Majorana spinors. The indices take the values $a = 0, 1, 2$ (Poincaré index), $\alpha = 1, 2$ (spin index) and $I = 1, \ldots, N$ (rigid $SO(N)$ index). The tangent space metric is Minkowski of signature is $(-++)$, the Levi-Civita tensor for the Poincaré indices $\epsilon^{abc} = \epsilon_{abc}$ is defined by $\epsilon^{123} = 1$ and the spin Levi-Civita tensor $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta}$ by $\epsilon^{12} = 1$. The latter is used in order to lower and raise the spin indices as follows: $u_\alpha = \mu^\beta \epsilon_{\beta\alpha}$, $u^\alpha = \epsilon^{\alpha\beta} u_\beta$. We also have $\epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = -\delta^\alpha_\beta$. The Dirac matrices $(\gamma^a_\alpha{}^\beta)$ are chosen real: $\gamma^0 = -\mathrm{i}\sigma_y$, $\gamma^1 = \sigma_z$, $\gamma^2 = \sigma_x$, the σ's being the Pauli matrices. In this representation the Majorana spinors have real components: $u^*_\alpha = u_\alpha$ and $\bar{u}^\alpha \equiv (u^+\gamma^0)^\alpha = \varepsilon^{\alpha\beta} u_\beta = u^\alpha$.

Let us collect in one array X_A the basis elements of the superalgebra:

$$
\{X_A\} = \{P_a,\, J_b,\, Q^I_\alpha\, ;\ a, b = 0, 1, 2\, ;\ \alpha = 1, 2\, ;\ I = 1, \ldots, N\} .
$$

In order to construct a Chern–Simons action, we need a non-degenerate invariant quadratic form

$$
\langle \Phi_1, \Phi_2 \rangle = g_{ab} \Phi_1^A \Phi_2^B \,, \quad \text{with } \Phi_{1,2} = \Phi^A_{1,2} X_A \,, \tag{2.2}
$$

invariant under the adjoint action of the superalgebra

$$
\delta_A \Phi_{1,2} = [\Phi_{1,2}, X_A] .
$$

Such a quadratic form may be derived from the following quadratic Casimir operator of the algebra (2.1):

$$
C = C^{AB} X_A X_B = P^a J_a - \frac{1}{4} Q^{I\alpha} Q^I_\alpha, \tag{2.3}
$$

namely

$$
\begin{aligned}
(g_{AB}) &= \frac{1}{2} (-1)^{[X_A]} \left(C^{-1}_{AB}\right) \\
&= \begin{pmatrix} 0 & (\delta_{ab}) & 0 \\ (\delta_{ab}) & 0 & 0 \\ 0 & 0 & (2\epsilon_{\alpha\beta}) \end{pmatrix},
\end{aligned}
\tag{2.4}
$$

where $[X_A] = 0, 1$ if X_A is an even, odd generator, respectively.

Writing the Lie algebra valued Yang–Mills connection as

$$
A = A^A X_A = e^a P_a + \omega^a J_a + \psi^{I\alpha} Q^I_\alpha \,, \tag{2.5}
$$

where e^a, ω^a and $\psi^{\alpha I}$ are the dreibein, spin connection and gravitino 1-forms, respectively, we can now write the Chern–Simons action as follows:

$$
\begin{aligned}
S_{\mathrm{CS}} &= \frac{1}{2} \int \langle A, \mathrm{d}A + A^2 \rangle \\
&= -\int \left[e^a \left(\mathrm{d}\omega_a + \frac{1}{2}\epsilon_{abc}\omega^b\omega^c \right) + \psi^I_\alpha \mathrm{d}\psi^{I\alpha} \right. \\
&\qquad \left. + \frac{1}{2}\omega^a \psi^{I\alpha}\psi^{I\beta}\gamma_{a\alpha\beta} \right] .
\end{aligned}
\tag{2.6}
$$

The symbol $\langle\ ,\ \rangle$ denotes the invariant quadratic form (2.2). This action is obviously invariant under rigid $SO(N)$ transformations, under which ψ transforms as a vector and the remaining fields as scalars. We may turn this invariance into a local $SO(N)$ invariance substituting the derivative $\mathrm{d}\psi^I$ through the covariant derivative $D_{\mathrm{O}}\psi^I = \mathrm{d}\psi^I + A^{IJ}_{\mathrm{O}}\psi^J$, where A^{IJ}_{O} is a non-dynamical $SO(N)$ connection.

The fields A^{IJ}_{O} then play the role of sources of the conserved Noether currents

$$
j^{IJ\mu} = \varepsilon^{\mu\nu\rho} \psi^I_{\alpha\nu} \psi^{J\alpha}_\rho
$$

of the $SO(N)$ symmetry. We may observe that the connection A_{O} cannot be made a dynamical field for the reason that a quadratic Casimir operator such as (2.3) containing the $SO(N)$ generators is not invertible, hence does not lead to the invariant quadratic form necessary for writing a kinetic action. This is in contrast with what happens in the super-anti-de Sitter case, where the relevant quadratic Casimir operator is indeed invertible [4,9].

The first term in (2.6) is the Einstein–Palatini action and the others are the kinetic term of the gravitino and its interaction with the spin connection.

The equations of motion derived from the action (2.6) read[2]

$$
\begin{aligned}
\frac{\delta S}{\delta e^a} &= \mathrm{d}\omega_a + \frac{1}{2}\epsilon_{abc}\omega^b\omega^c \stackrel{*}{=} 0\,, \quad \text{(Curvature)}, \\
\frac{\delta S}{\delta \omega^a} &= \mathrm{d}e_a + \epsilon_{abc} e^b \omega^c + \frac{1}{2}\psi^{I\alpha}\psi^{I\beta}\gamma_{a\alpha\beta} \stackrel{*}{=} 0\,, \\
&\qquad \text{(Torsion)}, \\
\frac{\delta S}{\delta \psi^I_\alpha} &= 2\left(\mathrm{d}\psi^{I\alpha} - \frac{1}{2}\omega^a \gamma^\alpha_{a\beta}\psi^{I\beta} \right) \stackrel{*}{=} 0\,. \\
&\qquad \text{(Rarita–Schwinger)}.
\end{aligned}
\tag{2.7}
$$

We see that the curvature is zero. The torsion equation gives us the spin connection as a function of the dreibein and the gravitino. The Rarita–Schwinger equation expresses the vanishing of the gravitino covariant derivative with respect to the gauge group $SO(1,2)$. The local symmetries of the action are expressed as its invariance under the BRST transformations

$$
sA = -\mathrm{d}C - [A, C], \quad sC = -C^2, \quad s^2 = 0\,, \tag{2.8}
$$

[1] The "graded bracket" (,) is an anticommutator if both entries are odd, and a commutator otherwise.

[2] The symbol $\stackrel{*}{=}$ means equal up to an equation of motion ("on-shell").

where the Faddeev–Popov ghost C is written in the adjoint representation of the Poincaré supergroup:

$$C = C^A X_A = c_{\rm T}^a P_a + c_{\rm L}^a J_a + c_{\rm S}^{I\alpha} Q_\alpha^I \,;$$

the fields $c_{\rm T}^a$, $c_{\rm L}^a$, $c_{\rm S}^{I\alpha}$ are the ghosts associated to space-time translations, Lorentz and supersymmetry transformations, respectively. They are 0-forms of ghost number 1 by definition. The ghosts $c_{\rm T}^a$, $c_{\rm L}^a$ are odd and $c_{\rm S}^{I\alpha}$ is even. In components, the BRST transformations read

$$\begin{aligned}
se^a &= -{\rm d}c_{\rm T}^a + \epsilon^a{}_{bc}\omega^c c_{\rm T}^b + \epsilon^a{}_{bc}e^c c_{\rm L}^b - \gamma^a_{\alpha\beta}\psi^{I\alpha}c_{\rm S}^{I\beta}\,,\\
s\omega^a &= -{\rm d}c_{\rm L}^a + \epsilon^a{}_{bc}\omega^c c_{\rm L}^b\,,\\
s\psi^{I\alpha} &= -{\rm d}c_{\rm S}^{I\alpha} + \frac{1}{2}\gamma_{a\beta}{}^{\alpha}\omega^a c_{\rm S}^{I\beta} + \frac{1}{2}\gamma_{a\beta}{}^{\alpha}\psi^{I\beta}c_{\rm L}^a\,,\\
sc_{\rm T}^a &= \epsilon^a{}_{bc}c_{\rm T}^c c_{\rm L}^b - \frac{1}{2}\gamma^a_{\alpha\beta}c_{\rm S}^{I\alpha}c_{\rm S}^{I\beta}\,,\\
sc_{\rm L}^a &= \frac{1}{2}\epsilon^a{}_{bc}c_{\rm L}^c c_{\rm L}^b\,,\\
sc_{\rm S}^{I\alpha} &= \frac{1}{2}\gamma_{a\beta}{}^{\alpha}c_{\rm L}^a c_{\rm S}^{I\beta}\,.
\end{aligned}\tag{2.9}$$

The BRST invariance can be expressed through a Slavnov–Taylor identity. We introduce the Batalin–Vilkovisky antifields A^* and C^* associated to A and C:

$$\begin{aligned}
A^* &= \omega^{*a}P_a + e^{*a}J_a + \psi^{*\alpha I}Q_\alpha^I\,,\\
C^* &= c_{\rm L}^{*a}P_a + c_{\rm T}^{*a}J_a + c_{\rm S}^{*I\alpha}Q_\alpha^I\,,
\end{aligned}\tag{2.10}$$

and an action coupling them to the BRST transformations of A and C:

$$\begin{aligned}
S_{\rm ext} &= \int (\langle A^*, sA\rangle + \langle C^*, sC\rangle)\\
&= \int \big(e^* se + \omega^* s\omega + \psi_\alpha^{*I} s\psi^{*\alpha I}\\
&\qquad + c_{\rm L}^* sc_{\rm L} + c_{\rm T}^* sc_{\rm T} + c_{\rm S}^{*I\alpha} sc^I_{{\rm S}\alpha}\big)\,.
\end{aligned}\tag{2.11}$$

The exterior field A^* is a 2-form of ghost number -1 and C^* a 3-form of ghost number -2. The Grassmann parities of the fields A, C, A^* and C^* are determined through their total degree: ghost number plus form degree. The field is even (commuting) or odd (anticommuting) if its total degree is even or odd, respectively.

The total action

$$S = S_{\rm CS} + S_{\rm ext} \tag{2.12}$$

obeys the Slavnov–Taylor identity

$$\mathcal{S}(S) = \int \sum_{\varphi = A,C} \frac{\delta S}{\delta\varphi^*}\frac{\delta S}{\delta\varphi} = 0. \tag{2.13}$$

We can work in a more compact way defining an extended field [8] $\tilde{A}$ written as the sum of forms of all possible degrees (in our case, degrees 0 to 3):

$$\tilde{A} = C + A + A^* + C^*\,, \tag{2.14}$$

the total degree of $\tilde{A}$ being equal 1. We also define an extended exterior derivative $\tilde{\rm d} = b + {\rm d}$, where b is a BRST type operator such that $b^2 = [b, {\rm d}] = 0$, and therefore $\tilde{\rm d}^2 = 0$. The extended zero curvature condition is

$$\tilde{F} = \tilde{\rm d}\tilde{A} + \tilde{A}^2 = 0. \tag{2.15}$$

This condition gives us the b transformation of the extended field

$$b\tilde{A} = -{\rm d}\tilde{A} - \tilde{A}^2\,, \tag{2.16}$$

and hence of its components C, A, C^* and A^*. The operator b can be interpreted as the linearized Slavnov–Taylor operator associated with an action $S(\varphi, \varphi^*)$:

$$b = \mathcal{S}_{\rm S} = \sum_{\varphi = A,C} \int \frac{\delta S}{\delta\varphi^*}\frac{\delta}{\delta\varphi} + \frac{\delta S}{\delta\varphi}\frac{\delta}{\delta\varphi^*}\,, \tag{2.17}$$

provided the action S is a solution of the equations

$$\frac{\delta S}{\delta\varphi^*} = \mathcal{S}_{\rm S}\varphi = b\varphi\,,\quad \frac{\delta S}{\delta\varphi} = \mathcal{S}_{\rm S}\varphi^* = b\varphi^* \quad (\varphi = C,\, A)\,. \tag{2.18}$$

The general solution of the latter equations is the action (2.12). This argument shows that one could have proceeded in a reversed way as well, namely beginning with the construction of a nilpotent operator b acting on the fields and antifields, and then deriving the action as a solution of (2.18). This is the procedure we shall follow in the next subsection. Note that the fulfilment of (2.18) automatically ensures [9] the validity of the Slavnov–Taylor identity (2.13).

2.1 Diffeomorphism and vector SUSY

As a topological one, our theory must be invariant under the diffeormorphisms or general coordinates transformations. As we want to include them in the BRST operator, we treat the infinitesimal diffeormorphism parameter as a ghost vector $\xi = \xi^\mu\partial_\mu$, the components ξ_μ being odd. The diffeormorphism transformation

$$\delta_{\rm diff}\,\varphi = \mathcal{L}_\xi\varphi\,,\quad \delta_{\rm diff}\,\varphi^* = \mathcal{L}_\xi\varphi^*\,, \tag{2.19}$$

where $\mathcal{L}_\xi$ is the Lie derivative associated with the vector field ξ, may thus be added to the BRST operator. We do it in the extended field formalism, still including the local vector supersymmetry transformation which uses to accompany diffeomorphism invariance [5]. The ghost of the latter is an even vector field $v = v^\mu\partial_\mu$ of ghost number 2 which happens to contribute to the BRST transformation of the diffeomorphism ghost ξ. In order to get this more general BRST operator, we define, as explained in the end of the last subsection, a nilpotent b-operator which reads, for the extended field and the new ghosts,

$$\begin{aligned}
b\tilde{A} &= -{\rm d}\tilde{A} - \tilde{A}^2 + \mathcal{L}_\xi\tilde{A} - i_v\tilde{A}\,,\\
b\xi &= \xi^2 + v\,,\quad bv = [\xi, v]\,.
\end{aligned}\tag{2.20}$$

For each form degree this gives

$$\begin{aligned} bC &= -C^2 + \mathcal{L}_\xi C - i_v A\,,\\ bA &= -\mathrm{d}C - [A,C] + \mathcal{L}_\xi A - i_v A^*\,,\\ bA^* &= -\mathrm{d}A - A^2 - [A^*,C] + \mathcal{L}_\xi A^* - i_v C^*\,,\\ bC^* &= -\mathrm{d}A^* - [A^*,A] - [C^*,C] + \mathcal{L}_\xi C^*\,,\\ b\xi &= \xi^2 + v\,,\\ bv &= [\xi, v] \end{aligned} \tag{2.21}$$

or, in components,

$$\begin{aligned} bc_{\mathrm{T}}^a &= \epsilon^a_{bc} c_{\mathrm{T}}^c c_{\mathrm{L}}^b - \frac{1}{2}\gamma^a_{\alpha\beta} c_{\mathrm{S}}^{I\alpha} c_{\mathrm{S}}^{I\beta} + \mathcal{L}_\xi c_{\mathrm{T}}^a - i_v e^a,\\ bc_{\mathrm{L}}^a &= \frac{1}{2}\epsilon^a_{bc} c_{\mathrm{L}}^c c_{\mathrm{L}}^b + \mathcal{L}_\xi c_{\mathrm{L}}^a - i_v \omega^a,\\ bc_{\mathrm{S}}^{I\alpha} &= \frac{1}{2}\gamma^\alpha_{a\beta} c_{\mathrm{L}}^a c_{\mathrm{S}}^{I\beta} + \mathcal{L}_\xi c_{\mathrm{S}}^{I\alpha} - i_v \psi^{I\alpha},\\ be^a &= -\mathrm{d}c_{\mathrm{T}}^a + \epsilon^a_{bc} e^c c_{\mathrm{L}}^b + \epsilon^a{}_{bc}\omega^c c_{\mathrm{T}}^b - \gamma^a_{\alpha\beta}\psi^{I\alpha} c_{\mathrm{S}}^{I\beta}\\ &\quad + \mathcal{L}_\xi e^a - i_v \omega^{*a},\\ b\omega^a &= -\mathrm{d}c_{\mathrm{L}}^a + \epsilon^a{}_{bc}\omega^c c_{\mathrm{L}}^b + \mathcal{L}_\xi \omega^a - i_v e^{*a},\\ b\psi^{I\alpha} &= -\mathrm{d}c_{\mathrm{S}}^{I\alpha} + \frac{1}{2}\gamma^\alpha_{a\beta}\omega^a c_{\mathrm{S}}^{I\beta} + \frac{1}{2}\gamma^\alpha_{a\beta}\psi^{I\beta} c_{\mathrm{L}}^a\\ &\quad + \mathcal{L}_\xi \psi^{I\alpha} - i_v \psi^{*I\alpha}, \end{aligned} \tag{2.22}$$

and, for the anti-fields,

$$\begin{aligned} be^{*a} &= -\mathrm{d}e^a + \epsilon^a_{bc} e^c \omega^b - \frac{1}{2}\gamma^a_{\alpha\beta}\psi^{I\alpha}\psi^{I\beta}\\ &\quad + \epsilon^a{}_{bc} e^{*c} c_{\mathrm{L}}^b + \epsilon^a_{bc}\omega^{*c} c_{\mathrm{T}}^b - \gamma^a_{\alpha\beta}\psi^{*I\alpha} c_{\mathrm{S}}^{I\beta}\\ &\quad + \mathcal{L}_\xi e^{*a} - i_v c_{\mathrm{T}}^{*a},\\ b\omega^{*a} &= -\mathrm{d}\omega^a + \frac{1}{2}\epsilon^a_{bc}\omega^c\omega^b + \epsilon^a{}_{bc}\omega^{*c} c_{\mathrm{L}}^b\\ &\quad + \mathcal{L}_\xi \omega^{*a} - i_v c_{\mathrm{L}}^{*a},\\ b\psi^{*I\alpha} &= -\mathrm{d}\psi^{I\alpha} + \frac{1}{2}\gamma^\alpha_{a\beta}\omega^a\psi^{I\beta} + \frac{1}{2}\gamma^\alpha_{a\beta}\omega^{*a} c_{\mathrm{S}}^{I\beta}\\ &\quad + \frac{1}{2}\gamma_{a\beta}{}^\alpha \psi^{*I\beta} c_{\mathrm{L}}^a + \mathcal{L}_\xi \psi^{*I\alpha} - i_v c_{\mathrm{S}}^{*I\alpha},\\ bc_{\mathrm{T}}^{*a} &= -\mathrm{d}e^{*a} + \epsilon^a{}_{bc} e^{*c}\omega^b + \epsilon^a_{bc}\omega^{*c} e^b\\ &\quad - \gamma^a_{\alpha\beta}\psi^{*I\alpha}\psi^{I\beta} + \epsilon^a{}_{bc} c_{\mathrm{T}}^{*c} c_{\mathrm{L}}^b + \epsilon^a_{bc} c_{\mathrm{L}}^{*c} c_{\mathrm{T}}^b\\ &\quad - \gamma^a_{\alpha\beta} c_{\mathrm{S}}^{*I\alpha} c_{\mathrm{S}}^{I\beta} + \mathcal{L}_\xi c_{\mathrm{T}}^{*a},\\ bc_{\mathrm{L}}^{*a} &= -\mathrm{d}\omega^{*a} + \epsilon^a{}_{bc}\omega^{*c}\omega^b + \epsilon^a{}_{bc} c_{\mathrm{L}}^{*c} c_{\mathrm{L}}^b + \mathcal{L}_\xi c_{\mathrm{L}}^{*a},\\ bc_{\mathrm{S}}^{*I\alpha} &= -\mathrm{d}\psi^{*I\alpha} + \frac{1}{2}\gamma_{a\beta}{}^\alpha \omega^{*a}\psi^{I\beta}\\ &\quad + \frac{1}{2}\gamma_{a\beta}{}^\alpha \psi^{*I\beta}\omega^a + \frac{1}{2}\gamma_{a\beta}{}^\alpha c_{\mathrm{L}}^{*a} c_{\mathrm{S}}^{I\beta}\\ &\quad + \frac{1}{2}\gamma_{a\beta}{}^\alpha \psi^{*I\beta} c_{\mathrm{L}}^a + \mathcal{L}_\xi c_{\mathrm{S}}^{*I\alpha}. \end{aligned} \tag{2.23}$$

Now the operator b can be interpreted as the following linearized Slavnov–Taylor operator associated with an action $S(\varphi,\varphi^*,v,\xi)$:

$$\mathcal{S}_S = \int \sum_{\varphi=A,C} \frac{\delta S}{\delta\varphi^*}\frac{\delta}{\delta\varphi} + \frac{\delta S}{\delta\varphi}\frac{\delta}{\delta\varphi^*} + \sum_{u=v,\xi} bu\,\frac{\delta}{\delta u}\,, \tag{2.24}$$

with the transformations bu explicitly[3] given in (2.21). Indeed, (2.18) are solved by the action

$$S(\varphi,\varphi^*,\xi,v) = -\frac{1}{2}\int \left\langle \tilde{A}, \mathrm{d}\tilde{A} + \frac{2}{3}\tilde{A}^2 - \mathcal{L}_\xi\tilde{A} + i_v\tilde{A}\right\rangle. \tag{2.25}$$

The integral of an extended form is defined as the integral of its 3-form terms. This action yields

$$\begin{aligned} S(\varphi,\varphi^*,\xi,v) = \int\Big(&-\frac{1}{2}\left\langle A,\, \mathrm{d}A + \frac{2}{3}A^2\right\rangle\\ &+ \langle A^*,\, -\mathrm{d}C - [A,C] + \mathcal{L}_\xi A\rangle\\ &+ \langle C^*, -C^2 + \mathcal{L}_\xi C - i_v A\rangle\\ &- \frac{1}{2}\langle A^*,\, i_v A^*\rangle\Big). \end{aligned} \tag{2.26}$$

2.2 Gauge fixing

The gauge invariant action obtained in the preceding subsection has still to be gauge fixed. We shall use the Batalin–Vilkovisky scheme [7,10]. The total action will then have the form

$$S = S_{\mathrm{CS}} + S_{\mathrm{ext}} + S_{\mathrm{gf}}\,. \tag{2.27}$$

In order to determine the gauge fixing part S_{gf}, we choose the Landau gauge condition, which necessitates the introduction of a non-dynamical background metric $g_{\mu\nu}$[4]. In differential form notation, the gauge condition reads

$$\mathrm{d} * A = \sqrt{g}\nabla_\mu A^\mu \mathrm{d}^3x\,,$$

where ∇_μ is the covariant derivative with respect to the background metric. It will be implemented through a Lautrup–Nakanishia Lagrange multiplier B and its associated Faddeev–Popov antighost $\bar{C}$, both being 0-forms and Lie algebra valued:

$$\begin{aligned} \bar{C} &= \bar{C}^A X_A = \bar{c}^a_{\mathrm{T}} P_a + \bar{c}^a_{\mathrm{L}} J_a + \bar{c}^{\alpha I}_{\mathrm{S}} Q^I_\alpha\,,\\ B &= B^A X_A = B^a_{\mathrm{T}} P_a + B^a_{\mathrm{L}} J_a + B^{\alpha I}_{\mathrm{S}} Q^I_\alpha\,, \end{aligned} \tag{2.28}$$

their BRST transformations being defined by

$$s\bar{C} = B\,,\quad sB = 0\,. \tag{2.29}$$

The total gauge fixed action $S(\varphi,\varphi^*,\xi,v,B,\bar{C})$ obeying the Slavnov–Taylor identity,

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[3] This means that we consider the ghost ξ and v as external fields.

[4] The dynamical metric is represented by the dreibein e^a

$$\mathcal{S}(S) = \sum_\varphi \int \left(\frac{\delta S}{\delta \varphi^*} \frac{\delta S}{\delta \varphi} + (\xi^2 + v) \frac{\delta S}{\delta \xi} + [\xi, v] \frac{\delta S}{\delta v} + B \frac{\delta S}{\delta \bar{C}} \right) = 0 \,, \tag{2.30}$$

is given, according to Batalin and Vilkovisky, by

$$S(\varphi, \varphi^*, B, \bar{C}) = S(\varphi, \hat{\varphi}^*) + \int B \mathrm{d} * A \,, \tag{2.31}$$

where the antifields φ^* in (2.26) have been replaced by

$$\hat{\varphi}^* = \varphi^* + \frac{\delta \Psi}{\delta \varphi} \,,$$

the "Batalin–Vilkovisky fermion", Ψ being a local functional of ghost number -1, chosen here as

$$\Psi = \Psi(\varphi, \bar{C}) = \int \mathrm{d}\bar{C} * A = \int A * \mathrm{d}\bar{C}. \tag{2.32}$$

We thus have

$$\hat{A}^* = A^* + * \mathrm{d}\bar{C} \,, \quad \hat{C}^* = C^* \,, \tag{2.33}$$

and the total gauge fixed action reads

$$\begin{aligned} S(\varphi, \varphi^*, \xi, v) = \int \Bigg(& -\frac{1}{2} \left\langle A, \, \mathrm{d}A + \frac{2}{3} A^2 \right\rangle \\ & + \left\langle \hat{A}^*, \, -\mathrm{d}C - [A, C] + \mathcal{L}_\xi A \right\rangle \\ & + \left\langle C^*, -C^2 + \mathcal{L}_\xi C - i_v A \right\rangle \\ & - \frac{1}{2} \left\langle \hat{A}^*, \, i_v \hat{A}^* \right\rangle \Bigg) \,. \end{aligned} \tag{2.34}$$

In terms of the components fields e^a, etc., we have

$$\begin{aligned} S = -\int \Bigg(& e^a \mathrm{d}\omega_a + \frac{1}{2} \epsilon_{abc} e^a \omega^b \omega^c + \psi^I_\alpha \mathrm{d}\psi^{I\alpha} \\ & - \frac{1}{2} \psi^I_\alpha \omega^a \psi^{I\beta} \gamma_{a\beta}{}^\alpha \\ & + \hat{e}^{*a} \big(-\mathrm{d}c_{\mathrm{T}a} + \epsilon_{abc} e^c c^b_{\mathrm{L}} + \epsilon_{abc} \omega^c c^b_{\mathrm{T}} \\ & \qquad - \gamma_{a\alpha\beta} \psi^{I\alpha} c^{I\beta}_{\mathrm{S}} + \mathcal{L}_\xi e_a \big) \\ & + \hat{\omega}^{*a} \left(-\mathrm{d}c_{\mathrm{L}a} + \epsilon_{abc} \omega^c c^b_{\mathrm{L}} + \mathcal{L}_\xi \omega_a \right) \\ & + \hat{\psi}^{*I}_\alpha \Bigg(-\mathrm{d}c^{I\alpha}_{\mathrm{S}} + \frac{1}{2} c^{I\beta}_{\mathrm{S}} \omega^a \gamma_{a\beta}{}^\alpha + \frac{1}{2} \gamma^{a\alpha}_\beta c_{\mathrm{L}a} \psi^{I\beta} \\ & \qquad + \mathcal{L}_\xi \psi^{I\alpha} \Bigg) \\ & + \hat{c}^{*a}_{\mathrm{T}} \Bigg(\epsilon_{abc} c^c_{\mathrm{T}} c^b_{\mathrm{L}} - \frac{1}{2} \gamma_{a\alpha\beta} c^{I\alpha}_{\mathrm{S}} c^{I\beta}_{\mathrm{S}} \\ & \qquad + \mathcal{L}_\xi c_{\mathrm{T}a} - i_v e_a \Bigg) \\ & + \hat{c}^{*a}_{\mathrm{L}} \left(\frac{1}{2} \epsilon_{abc} c^c_{\mathrm{L}} c^b_{\mathrm{L}} + \mathcal{L}_\xi c_{\mathrm{L}a} - i_v \omega_a \right) \\ & + \hat{c}^{*I}_{\mathrm{S}\alpha} \left(\frac{1}{2} \gamma^\alpha_{a\beta} c^a_{\mathrm{L}} c^{I\beta}_{\mathrm{S}} + \mathcal{L}_\xi c^{I\alpha}_{\mathrm{S}} - i_v \psi^{I\alpha} \right) \\ & - \frac{1}{2} \hat{\omega}^{*a} i_v \hat{e}^*_a - \frac{1}{2} \hat{e}^{*a} i_v \hat{\omega}^*_a - \frac{1}{2} \hat{\psi}^{*I}_\alpha i_v \hat{\psi}^{*\alpha I} \\ & + B^a_{\mathrm{T}} \mathrm{d} * \omega_a + B^a_{\mathrm{L}} \mathrm{d} * e_a + B^I_{\mathrm{S}\alpha} \mathrm{d} * \psi^{\alpha I} \Bigg), \end{aligned} \tag{2.35}$$

where

$$* \mathrm{d}B_{\mu\nu} = \frac{1}{2\sqrt{g}} \epsilon_{\mu\nu}{}^\rho \partial_\rho B \,, \tag{2.36}$$

and

$$\begin{aligned} \hat{e}^{*a} &= e^{*a} - * \mathrm{d}\bar{c}^a_{\mathrm{T}} \,, \quad \hat{\omega}^{*a} = \omega^{*a} - * \mathrm{d}\bar{c}^a_{\mathrm{L}} \,, \\ \hat{\psi}^{*I\alpha} &= \psi^{*I\alpha} - * \mathrm{d}\bar{c}^{I\alpha}_{\mathrm{S}} \,, \\ \hat{c}^{*a}_{\mathrm{T}} &= c^{*a}_{\mathrm{T}} \,, \quad \hat{c}^{*a}_{\mathrm{L}} = c^{*a}_{\mathrm{L}} \,, \quad \hat{c}^{*I\alpha}_{\mathrm{S}} = c^{*I\alpha}_{\mathrm{S}} \,. \end{aligned} \tag{2.37}$$

This is a gauge fixed action for 2 + 1-dimensional N-extended supergravity.

3 Conclusion

In this work we have achieved the gauge fixing in the Batalin–Vilkovisky scheme, of 2 + 1-dimensional N-extended supergravity considered as a topological field theory of the Chern–Simons type, the gauge group being the N-extended Poincaré supergroup. We have used the formalism of extended fields in order to express the BRST algebra, the Slavnov–Taylor identity and the action in a compact and manageable way.

It would be interesting to consider the construction in terms of a super-BF topological theory, with the Lorentz group as a gauge group. This approach would have the advantage of making possible a generalization to higher dimensions, the super B-field being then however subjected to appropriate constraints [6], as in the non-supersymmetric case (see e.g. [11]).

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